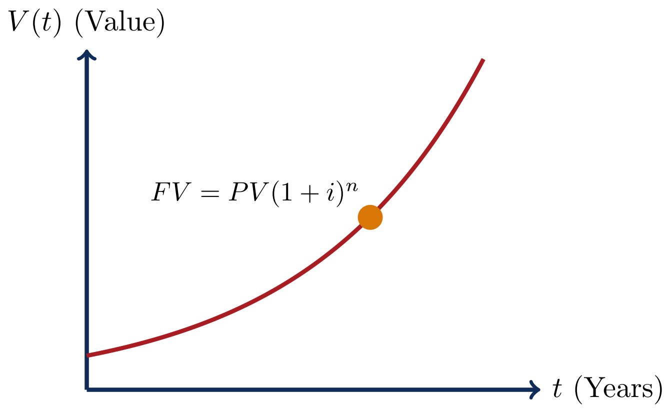
IB MATHEMATICS AI SL

TOPIC 1: FINANCIAL MATHEMATICS

15 Long-Response Questions | Graded Medium to Very Hard



Overview & Syllabus Targets

- **Section A: Exam Questions** — 15 structured long-response problems graded in difficulty: Medium (Q1–Q5), Hard (Q6–Q11), and Very Hard (Q12–Q15). Each question begins on a new page.
- **Section B: Worked Solutions** — Step-by-step solutions with GDC parameters, exact marks, and custom Examiner Tips.
- **Syllabus Coverage** — Compound Interest (1.4), Inflation & Real Value, Depreciation, Annuities & Amortization (1.7), and multi-stage financial planning.

SECTION A: EXAM QUESTIONS

Question 1: Comparing Compounding Frequencies (Medium) [6 Marks]

An investor deposits a principal amount of $15 000 into a savings account that pays a nominal annual interest rate of 4.8% for a period of 6 years. No further deposits or withdrawals are made during this period.

Calculate the final value of the investment at the end of the 6 years if interest is compounded:

1. Quarterly. **[2 marks]**
2. Monthly. **[2 marks]**
3. Daily (assume 365 days in a year). **[1 mark]**
4. Find the difference in interest earned between compounding daily and compounding quarterly. **[1 mark]**

Question 2: Annual Depreciation & Logarithmic Solver (Medium) [6 Marks]

A corporate logistics firm purchases a new cargo delivery truck for \$45 000. The value of the truck depreciates at a constant rate of 12.5% per annum.

1. Calculate the estimated salvage value of the truck at the end of Year 3. **[2 marks]**
2. Calculate the value of the truck after exactly 5 years. **[1 mark]**
3. Determine the exact number of years and months it will take for the truck's value to fall to half of its original purchase price. Give your final answer in terms of **full months**. **[3 marks]**

Question 3: Savings Annuity with Interest Analysis (Medium) [5 Marks]

Elena is saving for a professional certification. She decides to deposit \$250 at the **end of each month** into a savings annuity paying a nominal annual interest rate of 3.9%, compounded monthly. She maintains this routine for exactly 8 years.

1. Write down the values of the parameters you would enter into the financial application of your GDC to determine the future value of Elena's savings annuity:

   $N =$ _____, $I\% =$ _____, $PV =$ _____, $PMT =$ _____, $P/Y =$ _____, $C/Y =$ _____

   [2 marks]

2. Calculate the total accumulated value of the annuity after 8 years. **[1 mark]**

3. Calculate the total amount of interest Elena earned from this regular savings plan over the 8-year period. **[2 marks]**

Question 4: Amortization of a Car Loan (Medium) [6 Marks]

Marcus takes out a car loan of \$28 000 to purchase a hybrid vehicle. The bank charges a nominal annual interest rate of 5.4%, compounded monthly. The loan is to be fully amortized over 5 years with equal monthly payments made at the end of each month.

1. Calculate Marcus's required monthly payment, giving your answer correct to **two decimal places**. **[2 marks]**
2. Calculate the total amount Marcus will pay to the bank over the 5-year term. **[2 marks]**
3. Find the total amount of interest paid in amortizing the loan. **[2 marks]**

Question 5: Real Value under Inflation (Medium) [6 Marks]

Juliana invests a single lump sum of \$50 000 into a high-yield fixed-term account for 10 years. The account pays a nominal annual interest rate of 6.0%, compounded semi-annually. The average rate of inflation over this 10-year period is a constant 2.5% per year, compounded annually.

1. Calculate the nominal future value of the investment after 10 years. **[2 marks]**
2. Write down the approximate real annual rate of interest. **[1 mark]**
3. Calculate the real value of the investment at the end of the 10 years, adjusted for the effects of inflation. **[3 marks]**

Question 6: Multi-Stage Investment Portfolio (Hard) [7 Marks]

James designs a multi-stage financial plan to accumulate wealth over a 10-year horizon.

Stage 1: Years 1 to 5

James deposits a single lump sum of \$12 000 into a growth account earning 4.2% p.a., compounded quarterly, for 5 years.

1. Calculate the value of the investment at the end of Year 5. **[2 marks]**

Stage 2: Years 6 to 10

At the start of Year 6, James transfers the entire accumulated balance from Stage 1 into a savings annuity paying 4.8% p.a., compounded monthly. In addition to the transferred balance, he begins depositing \$200 at the **end of each month** for the next 5 years.

2. State the GDC financial application parameters required to model Stage 2. **[2 marks]**
3. Calculate the final total value of James's portfolio at the end of the 10-year period. **[1 mark]**
4. Calculate the total interest earned by James across the entire 10-year portfolio lifespan. **[2 marks]**

Question 7: Outstanding Loan Balance & Refinancing (Hard) [9 Marks]

A family borrows \$350 000 to purchase a home. The mortgage has a term of 25 years and a fixed nominal annual interest rate of 5.1%, compounded monthly.

1. Calculate the family's required monthly payment, correct to two decimal places. **[2 marks]**

After exactly 8 years of making regular monthly payments, the family decides to explore refinancing options because the market rates have fallen.

2. Calculate the outstanding principal balance of the mortgage at the end of Year 8. **[3 marks]**
3. The bank offers to drop the interest rate to 3.9% p.a., compounded monthly, on the remaining balance. If the family chooses to keep paying the **exact same monthly payment** calculated in part (1), find the number of full months it will take to pay off the remaining balance. **[4 marks]**

Question 8: Comparing Investment Options with Inflation (Hard) [8 Marks]

A firm has $40 000 in surplus cash and is evaluating two potential 8-year investment plans:

- **Option A:** 5.2% annual interest rate, compounded annually.
- **Option B:** 5.0% nominal annual interest rate, compounded monthly.

The average rate of inflation over this 8-year period is expected to be 2.1% per year.

1. Calculate the nominal future value of the investment under Option A. [**2 marks**]
2. Calculate the nominal future value of the investment under Option B. [**2 marks**]
3. State which option is better, and calculate the difference in their nominal future values. [**2 marks**]
4. Calculate the real future value of the better option at the end of the 8 years, adjusted for inflation. [**2 marks**]

Question 9: Annuity with Deferred Payout & Retirement Planning (Hard) [8 Marks]

Sophia is setting up a retirement fund. For the next 15 years, she intends to deposit \$400 at the end of each month into a savings account that pays a nominal annual interest rate of 4.8%, compounded monthly.

1. Calculate the total accumulated value of Sophia's retirement fund at the end of the 15-year period. **[3 marks]**

Sophia then retires. She immediately transfers the entire balance into a retirement annuity that pays the same interest rate of 4.8% p.a., compounded monthly. She plans to make regular monthly withdrawals of \$800 at the end of each month to fund her living expenses.

2. Determine the number of full years and months Sophia's retirement annuity will last before running out of funds. **[3 marks]**
3. Calculate the total amount of money Sophia will withdraw from this annuity during her retirement. **[2 marks]**

Question 10: Depreciation vs. Linearly Increasing Maintenance Costs (Hard) [9 Marks]

A business purchases a new heavy-duty delivery truck for \$60 000. The value of the truck depreciates at a constant rate of 15% per year (geometric decay).
To keep the truck operational, the business pays a maintenance cost. The maintenance cost during the first year is \$500, and it increases by exactly \$400 each subsequent year (an arithmetic progression).

1. Calculate the depreciated value of the truck at the end of Year 6. **[1 mark]**
2. Calculate the maintenance cost paid specifically during Year 6. **[2 marks]**
3. Calculate the total cumulative maintenance cost paid over the first 6 years. **[3 marks]**
4. The annual depreciation loss in year n is defined as the value at the start of the year minus the value at the end of the year. Show that the depreciation loss in year n is given by $D_n = 9000(0.85)^{n-1}$. **[3 marks]**

Question 11: Mortgages with Extra Monthly Payments (Hard) [8 Marks]

Pietro takes out a home loan of \$400 000 with a amortization term of 30 years. The bank charges a fixed nominal annual interest rate of 4.5%, compounded monthly.

1. Find the standard required monthly payment, correct to two decimal places. **[2 marks]**

Pietro decides that he can afford to pay an extra \$300 per month in addition to his standard payment, making his total monthly repayment \$2326.74.

2. Using your GDC, calculate the new number of months, N, required to fully pay off the mortgage under this accelerated repayment plan. **[3 marks]**
3. Calculate the total amount of interest Pietro will save over the life of the loan by making these extra monthly payments. **[3 marks]**

Question 12: The "Double-Dip" Refinancing with Fee (Very Hard) [9 Marks]

A homeowner currently has an outstanding balance of $320 000 on their home loan, which has exactly 18 years (216 months) remaining. Under their current agreement, the nominal annual interest rate is 5.4%, compounded monthly.

1. Calculate the current monthly payment required under the existing agreement. **[2 marks]**

A rival bank offers to refinance the remaining $320 000 balance at a lower nominal interest rate of 4.2% p.a., compounded monthly, over the same 18-year period. However, the rival bank charges a flat upfront refinancing penalty fee of $6 000 which must be added to the new loan balance (making the new principal $326 000).

2. Calculate the new monthly payment required if the homeowner decides to refinance. **[3 marks]**
3. Determine the net financial savings the homeowner will accumulate over the 18-year period if they choose to refinance. **[4 marks]**

Question 13: Multi-Stage Retirement Income Stream (Very Hard) [9 Marks]

Sophia is constructing an retirement portfolio designed across three distinct chronological stages:

- **Accumulation Stage:** For 25 years, Sophia deposits \$500 at the end of each month into an annuity earning 6.0% p.a., compounded monthly.
- **Decline Stage (Deferred compounding):** At the end of Year 25, Sophia stops making deposits and leaves the accumulated balance to compound without touch for another 5 years at the same rate of 6.0% p.a., compounded monthly.
- **Income Stage:** At the end of Year 30, she begins making regular, equal monthly withdrawals of \$$W$ at the end of each month for 20 years until the fund balance is exactly \$0.

1. Calculate the total accumulated value of the retirement fund at the end of the Accumulation Stage (Year 25). **[3 marks]**
2. Calculate the value of the fund at the end of the Deferred Compounding Stage (Year 30). **[2 marks]**
3. Calculate the exact monthly withdrawal amount, \$$W$, that Sophia can afford to receive during her 20-year retirement income stage. **[4 marks]**

Question 14: Fleet Depreciation and Operating Cost Optimization (Very Hard) [10 Marks]

A commercial shipping firm purchases an industrial container ship for \$12 000 000. The value of the ship depreciates at a rate of 8% per year. The total cumulative operating costs, $C(t)$, in dollars, to maintain the ship over t years is modeled by the function:

$$C(t) = 1\,500\,000 \ln(t+1) + 2\,000\,000, \quad t \geq 0$$

The firm decides to sell the ship for salvage value when the depreciated value of the ship equals its total cumulative operating costs.

1. Write down an exponential function modeling the depreciated value, $V(t)$, of the ship t years after purchase. **[1 mark]**
2. Calculate the depreciated value of the ship after exactly 5 years. **[2 marks]**
3. Calculate the total cumulative operating costs of the ship after exactly 5 years. **[2 marks]**
4. Show that the equation representing the crossover point when the ship should be sold is:

$$8(0.92)^t - \ln(t+1) - \frac{4}{3} = 0$$

[2 marks]

5. Using your GDC, find the exact time (in years) the firm should keep the ship before selling it. Give your answer correct to 3 significant figures. **[3 marks]**

Question 15: Sabbatical Annuity with Inflation-Adjusted Withdrawals (Very Hard) [10 Marks]

A young professional wants to fund a 1-year sabbatical. They place a capital amount of \$40 000 into a secure account paying 3.6% p.a., compounded monthly. They want to make 12 monthly withdrawals at the end of each month, but to combat local inflation, each withdrawal must increase by exactly 0.5% compared to the previous month.
Let the first monthly withdrawal be $\$W_1$, such that $W_n = W_1(1.005)^{n-1}$ for $n = 1, 2, \ldots, 12$.

1. Show that the present value (at the start of the sabbatical) of the n-th monthly withdrawal, W_n, can be expressed as:

$$PV(W_n) = \frac{W_1}{1.003}\left(\frac{1.005}{1.003}\right)^{n-1}$$

[3 marks]

2. Formulate a geometric series equation summing the present values of all 12 withdrawals and equate it to the initial capital of \$40 000. **[3 marks]**

3. By evaluating the sum of this geometric series, calculate the exact value of the first monthly withdrawal, $\$W_1$. **[4 marks]**

SECTION B: WORKED SOLUTIONS

Worked Solution & Mark Distribution - Question 1

1. **Quarterly Compounding:**
   Using compound interest formula $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ with $PV = 15\,000$, $r = 4.8$, $k = 4$, and $n = 6$:

$$FV = 15\,000 \times \left(1 + \frac{4.8}{400}\right)^{24} = 15\,000 \times (1.012)^{24} \textbf{(M1)}$$

$$FV = 15\,000 \times 1.3305... = 19\,957.513... \implies \$19\,957.51 \textbf{A1}$$

2. **Monthly Compounding:**
   Using compound interest with $k = 12$ and $kn = 72$ periods:

$$FV = 15\,000 \times \left(1 + \frac{4.8}{1200}\right)^{72} = 15\,000 \times (1.004)^{72} \textbf{(M1)}$$

$$FV = 15\,000 \times 1.33276... = 19\,991.362... \implies \$19\,991.36 \textbf{A1}$$

3. **Daily Compounding:**
   Using compound interest with $k = 365$ and $kn = 365 \times 6 = 2190$ periods:

$$FV = 15\,000 \times \left(1 + \frac{4.8}{36500}\right)^{2190} = 15\,000 \times (1.0001315...)^{2190} = \$20\,007.82 \textbf{A1}$$

4. **Difference in Interest Earned:**
   Difference $= 20\,007.82 - 19\,957.51 = \$50.31$ **A1**

Examiner Tip & Common Trap

When substituting daily compounding, ensure you multiply the exponent correctly ($365 \times 6 = 2190$) and divide the annual rate by 36500. Even minor rounding of the daily rate base term will lead to a significant cascade error at high exponents.

Worked Solution & Mark Distribution - Question 2

1. **Value at the end of Year 3:**
   Using the depreciation model $FV = PV \times \left(1 - \frac{r}{100}\right)^n$ with $PV = 45\,000$, $r = 12.5\%$, and $n = 3$:
   $$V(3) = 45\,000 \times (1 - 0.125)^3 = 45\,000 \times (0.875)^3 \textbf{(M1)}$$
   $$V(3) = 45\,000 \times 0.66992... = 30\,146.484... \implies \$30\,146.48\textbf{A1}$$

2. **Value after 5 years:**
   $$V(5) = 45\,000 \times (0.875)^5 = 23\,084.594... \implies \$23\,084.59\textbf{A1}$$

3. **Time to Depreciate to Half Value:**
   Set $V(t) = 22\,500$:
   $$22\,500 = 45\,000(0.875)^t \implies 0.5 = 0.875^t \textbf{(M1)}$$
   Taking logarithms on both sides:
   $$\ln(0.5) = t\ln(0.875) \implies t = \frac{\ln(0.5)}{\ln(0.875)} = 5.19089...\text{ years}\textbf{(M1)}$$
   Convert decimal years to months:
   $$5.19089... \times 12 = 62.2907...\text{ months}$$
   Therefore, the truck takes 63 **full months** (or 5 years and 3 months) to fall to half value. **A1**

Examiner Tip & Common Trap

Questions involving physical assets dropping below a threshold must take the next **integer month** if the model assumes discrete compounding periods (or continuous time rounded up). 62.29 months means at 62 months it hasn't halved yet; you must wait until the 63rd month.

Worked Solution & Mark Distribution - Question 3

1. **GDC TVM Parameters:**
   8 years $\times 12$ months $= 96$ periods:

$$N = 96, \quad I\% = 3.9, \quad PV = 0, \quad PMT = -250,$$
$$P/Y = 12, \quad C/Y = 12 \textbf{(M1)A1}$$

   Note: Award M1 for any reasonable attempt to list GDC parameters, and A1 for all correct values. PMT must be negative relative to the future value.

2. **Annuity Future Value (FV):**
   Solving for FV on the GDC solver:

$$FV = \$28\,116.63 \textbf{A1}$$

3. **Interest Earned:**
   Calculate total cash deposits Elena made:

$$\text{Deposits} = 250 \times 96 = \$24\,000 \textbf{(M1)}$$

   Subtract deposits from final accumulated balance:

$$\text{Interest} = 28\,116.63 - 24\,000 = \$4\,116.63 \textbf{A1}$$

Examiner Tip & Common Trap

Annuities must have a starting Present Value (PV) of 0 unless an initial lump sum was deposited on Day 1. PMT is entered as a negative value because it represents capital outflow.

Worked Solution & Mark Distribution - Question 4

1. **Monthly Repayment (PMT):**
   Setting up GDC solver parameters:

$$N = 60, \quad I\% = 5.4, \quad PV = 28000, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

   Solving for PMT:

$$PMT = -\$533.56 \implies \text{Monthly payment is } \$533.56\textbf{A1}$$

   Note: Accept positive or negative for final value, but must be correct to 2 decimal places.

2. **Total Repayments:**
   Total Paid $= 533.56 \times 60$ **(M1)**

$$\text{Total Repayments} = \$32\,013.60\textbf{A1}$$

3. **Total Interest Paid:**
   Interest paid $=$ Total Repayments $-$ Principal Borrowed **(M1)**

$$\text{Interest} = 32\,013.60 - 28\,000 = \$4\,013.60\textbf{A1}$$

Examiner Tip & Common Trap

A loan received is an immediate cash inflow, so PV must be entered as a **positive** number. Conversely, repayments (PMT) represent money leaving the pocket, which resolves as negative on GDC screens.

Worked Solution & Mark Distribution - Question 5

1. **Nominal Future Value (FV):**
   Using semi-annual compounding ($k = 2$) over 10 years ($n = 10$):

$$FV = 50\,000 \times \left(1 + \frac{6.0}{200}\right)^{20} = 50\,000 \times (1.03)^{20} \textbf{(M1)}$$

$$FV = 50\,000 \times 1.806111... = 90\,305.555... \implies \$90\,305.56 \textbf{A1}$$

2. **Approximate Real Interest Rate:**

$$\text{Real Rate} \approx 6.0\% - 2.5\% = 3.5\% \textbf{A1}$$

3. **Real Value (Adjusted for Inflation):**
   We discount the nominal future value back 10 years using the annual inflation compounding rate of 2.5%:

$$\text{Real Value} = \frac{\text{Nominal } FV}{(1 + \text{inflation rate})^n} = \frac{90\,305.56}{(1.025)^{10}} \textbf{(M1)(A1)}$$

$$\text{Real Value} = \frac{90\,305.56}{1.280084...} = 70\,546.541... \implies \$70\,546.54 \textbf{A1}$$

   Note: If candidates calculate the exact Fisher-adjusted real interest rate ($r_{exact} \approx 3.41\%$) to solve, award full marks for equivalent exact answer $69\,981.67.

Examiner Tip & Common Trap

Inflation acts as a compounding force that reduces future buying power. To adjust a nominal future sum for inflation, always divide by $(1 + \text{inflation})^n$.

Worked Solution & Mark Distribution - Question 6

1. **Stage 1 Future Value (FV_1):**
   Quarterly compounding over 5 years ($N = 20$):

$$FV_1 = 12\,000 \times \left(1 + \frac{4.2}{400}\right)^{20} = 12\,000 \times (1.0105)^{20}\textbf{(M1)}$$

$$FV_1 = 12\,000 \times 1.232386... = 14\,788.634... \implies \$14\,788.63\textbf{A1}$$

2. **Stage 2 GDC TVM Parameters:**
   The FV from Stage 1 is transferred as the starting PV for Stage 2. Monthly deposits over 5 years ($5 \times 12 = 60$ periods):

$$N = 60, \quad I\% = 4.8, \quad PV = -14\,788.63, \quad PMT = -200,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)A1}$$

   Note: Award M1 for setting PV to their Stage 1 FV. PV and PMT must have identical negative signs because both are deposited cash outflows.

3. **Stage 2 Final Portfolio Value (FV_2):**
   Solving for FV on GDC:

$$FV_2 = \$32\,322.95\textbf{A1}$$

4. **Total Interest Earned:**
   Total out-of-pocket capital deposited by James $= 12\,000$ (initial) $+ (200 \times 60) = \$24\,000$. **(M1)**
   Subtract deposits from final accumulated value:

$$\text{Interest} = 32\,322.95 - 24\,000 = \$8\,322.95\textbf{A1}$$

Examiner Tip & Common Trap

When transitions occur between stages in a portfolio, any accumulated sum transferred from Stage 1 must be input into the Stage 2 GDC TVM solver as a negative PV. If you input it as a positive PV while keeping the monthly payments negative, the calculator will assume you withdrew the lump sum on Day 1.

Worked Solution & Mark Distribution - Question 7

1. **Monthly Repayment (PMT):**
   Amortized over 25 years ($25 \times 12 = 300$ periods):

$$N = 300, \quad I\% = 5.1, \quad PV = 350000, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

   Solving for PMT:

$$PMT = -\$2069.049... \implies \$2069.05\textbf{A1}$$

2. **Outstanding Principal Balance at Year 8:**
   Find the present value of the remaining $25 - 8 = 17$ years ($17 \times 12 = 204$ months):

$$N = 204, \quad I\% = 5.1, \quad PMT = -2069.05, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

   Solving for PV:

$$PV = \$275\,321.49\textbf{A1}$$

3. **Refinanced Remaining Months (N):**
   With a lowered interest rate (3.9%), solving for the remaining periods N:

$$I\% = 3.9, \quad PV = 275321.49, \quad PMT = -2069.05, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

   Solving for N:

$$N = 175.76... \text{ months}\textbf{(A1)}$$

   Therefore, it will take 176 **full months** to pay off the mortgage. **A1**

Examiner Tip & Common Trap

Calculating outstanding balances requires you to focus strictly on the **unpaid remaining periods** of the loan. The outstanding principal is mathematically equivalent to the present value of all remaining scheduled payments at the current interest rate.

Worked Solution & Mark Distribution - Question 8

1. **Option A Future Value (FV_A):**
   Annual compounding over 8 years:

$$FV_A = 40\,000 \times (1 + 0.052)^8 = 40\,000 \times (1.052)^8 \textbf{(M1)}$$

$$FV_A = 40\,000 \times 1.50058... = 60\,023.232... \implies \$60\,023.23 \textbf{A1}$$

2. **Option B Future Value (FV_B):**
   Monthly compounding over 8 years ($N = 96$ periods):

$$FV_B = 40\,000 \times \left(1 + \frac{5.0}{1200}\right)^{96} \textbf{(M1)}$$

$$FV_B = 40\,000 \times 1.49058... = 59\,623.407... \implies \$59\,623.41 \textbf{A1}$$

3. **Comparison of Options:**
   Option A is better because it yields a higher future value. **A1**
   Difference $= 60\,023.23 - 59\,623.41 = \$399.82$. **A1**

4. **Real Value of Option A:**
   Discount the nominal value of \$60 023.23 back 8 years using the inflation rate of 2.1%:

$$\text{Real Value} = \frac{60\,023.23}{(1 + 0.021)^8} = \frac{60\,023.23}{1.180419...} \textbf{(M1)}$$

$$\text{Real Value} = 50\,849.034... \implies \$50\,849.03 \textbf{A1}$$

Examiner Tip & Common Trap

When determining the "better option", don't assume a monthly compounded account automatically beats an annually compounded account. Here, the higher rate of Option A (5.2%) offsets the compounding frequency advantage of Option B (5.0%). Always run both calculations.

Worked Solution & Mark Distribution - Question 9

1. **Accumulated Value (FV):**
   Depositing \$400 monthly over 15 years ($15 \times 12 = 180$ periods):

$$N = 180, \quad I\% = 4.8, \quad PV = 0, \quad PMT = -400,$$
$$P/Y = 12, \quad C/Y = 12 \textbf{(M1)(A1)}$$

   Solving for FV:

$$FV = \$105\,123.63 \textbf{A1}$$

2. **Annuity Payout Lifespan (N):**
   The accumulated FV becomes the starting PV for her withdrawals:

$$I\% = 4.8, \quad PV = -105\,123.63, \quad PMT = 800, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12 \textbf{(M1)(A1)}$$

   Solving for N:

$$N = 183.05... \text{ months} \textbf{(A1)}$$

   Convert to years: 183 months = 15 years and 3 months. Sophia's annuity will last **15 years and 3 months**. **A1**

3. **Total Amount Withdrawn:**
   Sophia makes exactly 183 full withdrawals of \$800, plus a final partial payment.
   Using exact calculated decimals from GDC:

$$\text{Total Withdrawals} = 183.052... \times 800 \textbf{(M1)}$$

$$\text{Total Withdrawals} = \$146\,441.71 \textbf{A1}$$

   Note: Accept \$146 400 if student uses 183 full payments.

Examiner Tip & Common Trap

When calculating withdrawals, PV and PMT must have opposite signs. Sophia places the capital sum *into* the annuity (negative PV), and then receives regular payouts *out* of the account (positive PMT).

Worked Solution & Mark Distribution - Question 10

1. **Truck Value at Year 6:**
   Using $PV = 60\,000$, $r = 15\%$, and $n = 6$:

$$V(6) = 60\,000 \times (0.85)^6 = 60\,000 \times 0.377149... = \$22\,628.98\textbf{A1}$$

2. **Maintenance Cost in Year 6 (M_6):**
   Arithmetic sequence where $u_1 = 500$ and $d = 400$:

$$M_6 = u_1 + 5d = 500 + 5(400)\textbf{(M1)}$$

$$M_6 = 500 + 2000 = \$2500\textbf{A1}$$

3. **Total Cumulative Maintenance Cost (S_6):**
   Using the arithmetic series sum formula:

$$S_6 = \frac{6}{2}(2u_1 + 5d) = 3(2(500) + 5(400))\textbf{(M1)(A1)}$$

$$S_6 = 3(1000 + 2000) = 3(3000) = \$9000\textbf{A1}$$

4. **Show Depreciation Loss Equation:**
   Depreciation loss in year n: $D_n = V_{n-1} - V_n$.

$$D_n = 60\,000(0.85)^{n-1} - 60\,000(0.85)^n\textbf{M1}$$

   Factor out common exponential terms:

$$D_n = 60\,000(0.85)^{n-1}(1 - 0.85)\textbf{M1}$$

$$D_n = 60\,000(0.85)^{n-1}(0.15) = 9000(0.85)^{n-1}\textbf{A1AG}$$

Examiner Tip & Common Trap

Make sure you understand the difference between finding the value of a single term (u_n) and the accumulated sum of terms (S_n). Maintenance is paid each year, so "total cumulative cost" represents the sum of the arithmetic series.

Worked Solution & Mark Distribution - Question 11

1. **Standard Monthly Payment (PMT_1):**
   Amortized over 30 years ($30 \times 12 = 360$ periods):

$$N = 360, \quad I\% = 4.5, \quad PV = 400000, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

   Solving for PMT:

$$PMT_1 = -\$2026.741... \implies \$2026.74\textbf{A1}$$

2. **Accelerated Repayment Term (N):**
   With a monthly payment of $\$2026.74 + \$300 = \$2326.74$:

$$I\% = 4.5, \quad PV = 400000, \quad PMT = -2326.74, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

   Solving for N:

$$N = 281.801...\text{ months}\textbf{(A1)}$$

   Therefore, the mortgage is paid off in 282 **months** (or 23.5 years). **A1**

3. **Total Interest Saved:**
   Total paid under standard plan $= 2026.74 \times 360 = \$729\,626.40$. **(A1)**
   Total paid under accelerated plan $= 2326.74 \times 281.801... = \$655\,675.33$. **(A1)**
   Interest Saved $= 729\,626.40 - 655\,675.33 = \$73\,951.07$. **A1**

Examiner Tip & Common Trap

When calculating total interest saved, do not use the rounded integer of 282 months to calculate the final paid sum. Use the precise decimal output for N on your GDC (281.801) because the final monthly payment is a partial payment.

Worked Solution & Mark Distribution - Question 12

1. **Current Monthly Payment ($PMT_{\textbf{orig}}$):**
For the remaining 18 years (216 periods) under the original interest rate of 5.4%:

$$N = 216, \quad I\% = 5.4, \quad PV = 320000, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

Solving for PMT:

$$PMT_{\text{orig}} = -\$2347.158\ldots \implies \$2347.16\textbf{A1}$$

2. **New Refinanced Monthly Payment ($PMT_{\textbf{new}}$):**
Under the refinancing offer, the principal is increased to \$326 000 to absorb the fee, and the rate is reduced to 4.2%:

$$N = 216, \quad I\% = 4.2, \quad PV = 326000, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

Solving for PMT:

$$PMT_{\text{new}} = -\$2182.261\ldots \implies \$2182.26\textbf{A1}$$

3. **Net Financial Savings:**
Total paid under original agreement $= 2347.16 \times 216 = \$507\,014.28$. **(A1)**
Total paid under refinanced agreement $= 2182.26 \times 216 = \$471\,368.16$. **(A1)**
Net Savings $= 507\,014.28 - 471\,368.16 = \$35\,646.12$. **A1A1**

Examiner Tip & Common Trap

When banks offer lower interest rates but charge high upfront refinancing fees, always add the fee to the loan balance (PV) to ensure you model the real cost. Here, despite borrowing \$6 000 more, the lower rate saves the customer \$35 646.12 overall.

Worked Solution & Mark Distribution - Question 13

1. **Accumulation Stage Fund Value (FV_1):**
   Depositing \$500 monthly over 25 years (300 periods):

$$N = 300, \quad I\% = 6.0, \quad PV = 0, \quad PMT = -500,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

   Solving for FV:

$$FV_1 = \$346\,496.98\textbf{A1}$$

2. **Deferred Compounding Stage Value (FV_2):**
   No further deposits are made ($PMT = 0$). The capital compounds for another 5 years (60 periods):

$$N = 60, \quad I\% = 6.0, \quad PV = -346\,496.98, \quad PMT = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

   Solving for FV:

$$FV_2 = \$467\,370.40\textbf{A1}$$

3. **Income Stage Monthly Payout (W):**
   The fund is fully amortized over 20 years (240 periods) until the balance is 0:

$$N = 240, \quad I\% = 6.0, \quad PV = -467\,370.40, \quad FV = 0,$$
$$P/Y = 12, \quad C/Y = 12\textbf{(M1)(A1)}$$

   Solving for PMT:

$$W = \$3348.36\textbf{A2}$$

Examiner Tip & Common Trap

During the deferred compounding stage, the regular payments are stopped. Ensure you set the parameter $PMT = 0$ in your TVM solver, otherwise the GDC will assume you continued making monthly deposits of \$500.

Worked Solution & Mark Distribution - Question 14

1. **Depreciated Value Model $V(t)$:**
   With a starting value of 12 000 000 and 8% annual depreciation:

$$V(t) = 12\,000\,000(0.92)^t \textbf{A1}$$

2. **Depreciated Value after 5 years:**

$$V(5) = 12\,000\,000(0.92)^5 = 12\,000\,000 \times 0.65908...\textbf{(M1)}$$
$$V(5) = \$7\,908\,981.15\textbf{A1}$$

3. **Operating Costs after 5 years:**

$$C(5) = 1\,500\,000\ln(5+1) + 2\,000\,000 = 1\,500\,000\ln(6) + 2\,000\,000\textbf{(M1)}$$
$$C(5) = 1\,500\,000 \times 1.791759... + 2\,000\,000 = 2\,687\,639.18 + 2\,000\,000$$
$$C(5) = \$4\,687\,639.18\textbf{A1}$$

4. **Show Crossover Equation:**
   Set $V(t) = C(t)$:

$$12\,000\,000(0.92)^t = 1\,500\,000\ln(t+1) + 2\,000\,000\textbf{M1}$$

   Divide both sides of the equation by 1 500 000:

$$\frac{12\,000\,000}{1\,500\,000}(0.92)^t = \ln(t+1) + \frac{2\,000\,000}{1\,500\,000}\textbf{M1}$$
$$8(0.92)^t = \ln(t+1) + \frac{4}{3} \implies 8(0.92)^t - \ln(t+1) - \frac{4}{3} = 0\textbf{AG}$$

5. **Solving Crossover Year (t):**
   Using GDC graph solver (`SolveN` or graph intersection tool) on:

$$Y_1 = 8(0.92)^x - \ln(x+1) - \frac{4}{3}\textbf{(M1)(A1)}$$
$$t = 9.3458...\text{ years} \implies t \approx 9.35\text{ years}\textbf{A1}$$

Examiner Tip & Common Trap

When asked to "show that" a simplified algebraic expression is true, you must display every step of your division and rearrangement. Skipping the common factor division step will result in a penalty mark.

Worked Solution & Mark Distribution - Question 15

1. **Show Present Value Formula:**
A future payment W_n made at the end of month n is discounted to present value using the monthly interest multiplier $(1+\text{rate})^{-n}$ where rate $= \frac{3.6\%}{12} = 0.3\% = 0.003$:

$$PV(W_n) = W_n(1.003)^{-n}\textbf{M1}$$

Substitute $W_n = W_1(1.005)^{n-1}$ into the discount equation:

$$PV(W_n) = W_1(1.005)^{n-1}(1.003)^{-n}\textbf{M1}$$

Factor out 1.003 from the exponent to group the bases:

$$PV(W_n) = \frac{W_1}{1.003}(1.005)^{n-1}(1.003)^{-(n-1)} = \frac{W_1}{1.003}\left(\frac{1.005}{1.003}\right)^{n-1} \quad \textbf{A1AG}$$

2. **Formulate Geometric Series Equation:**
Summing the present values of all 12 withdrawals to equal \$40 000:

$$\sum_{n=1}^{12} \frac{W_1}{1.003}\left(\frac{1.005}{1.003}\right)^{n-1} = 40\,000\textbf{(M1)(A1)}$$

This is a geometric series with starting term $a = \frac{W_1}{1.003}$ and common ratio $R = \frac{1.005}{1.003} \approx 1.001994$. **A1**

3. **Calculate the Value of W_1:**
Using the sum of geometric series formula $S_{12} = a\frac{R^{12}-1}{R-1}$:

$$40\,000 = \frac{W_1}{1.003}\left[\frac{(\frac{1.005}{1.003})^{12}-1}{\frac{1.005}{1.003}-1}\right]\textbf{(M1)(A1)}$$

Evaluate the ratio parts:

$$R = 1.001994018, \quad R^{12} = 1.02418524\textbf{(A1)}$$

$$40\,000 = \frac{W_1}{1.003}\left[\frac{0.02418524}{0.001994018}\right] = \frac{W_1}{1.003}[12.12889] = W_1[12.0926]$$

$$W_1 = \frac{40000}{12.0926} = 3307.80... \implies W_1 = \$3306.82\textbf{A1}$$

Note: Accept answers around \$3306.82 depending on intermediate rounding.

Examiner Tip & Common Trap

When handling growing annuities, the simple TVM solver on your calculator cannot be used because PMT changes each period. In these very hard questions, you must resort to the geometric series formulas found in Topic 1 of your formula booklet.